2017 IEEE 56th Annual Conference on Decision and Control (CDC)
December 12-15, 2017, Melbourne, Australia

Model-free tuning of plants with parasitic dynamics

Franco Blanchini [a], Gianfranco Fenu [b], Giulia Giordano [c], Felice Andrea Pellegrino [b]

***Abstract*— We have recently considered the problem of tuning a static plant described by a differentiable input-output function, which is completely unknown, but whose Jacobian takes values in a known polytope of matrices: to drive the output to a given desired value, we have suggested an integral feedback scheme, whose convergence is ensured if the polytope of matrices is robustly full row rank. The suggested tuning scheme may fail in the presence of parasitic dynamics, which may destabilize the loop if the tuning action is too aggressive. Here we show that such tuning action can be applied to dynamic plants as well if it is sufficiently "slow", a property that we can ensure by limiting the integral action. We provide robust bounds based on the exclusive knowledge of the largest time constant and of the matrix polytope to which the system Jacobian is known to belong. We also provide similar bounds in the presence of parasitic dynamics affecting the actuators.**

I. Introduction

For several types of systems with a large number of inputs and outputs (such as electrical networks, power generation systems, electronic circuits, systems for heat generation and transmission, flow networks in general), stability is not a critical issue, while steady-state tuning is very important and, at the same time, difficult to achieve. In fact, the plant model is often unknown, hence tuning requires a frustrating trial-and-error approach: when attempting to set an output to the desired value, the unknown interactions among the variables can unpredictably drive the other outputs out of tune.

In our recent papers [5], [6], we consider the problem of tuning a *static* plant, described by a system of nonlinear equations: the inputs of the plant need to be chosen so as to drive the outputs to the desired level, yet the system equations are unknown and only qualitative information on the system Jacobian is available. If the Jacobian matrix of the input-output function is included in a compact and convex set of matrices and such a set is robustly full row rank, the robust tuning problem can be solved by means of a proper tuning law. The result is based on the exploitation of a min-max theorem [16] that is well known in game theory, along with a Lyapunov-like function. The min-max theorem has been exploited in the context of robust control via Lyapunov methods since [12], [13] (see also [3], [8], [11], [17], [19]). The tuning algorithm provided in [5], [6] requires the solution of a convex optimization problem on-line.

In the discrete-time version of the problem, the min-max theorem cannot be applied. However, a solution is still possible [7] when the bounding set for the system Jacobian is a polytope, and still the on-line solution of a convex optimization problem is required.

The problem we consider bears resemblance to iterative learning control techniques [1], [9], which aim at determining the input function of a dynamic system so that the output function matches a desired reference: in principle, the scheme proposed in [5], [6] could be seen as an iterative (continuous-time) learning process for a static nonlinear plant. The technique is related to other methods previously adopted for parameter tuning [2], [10], in which however the goal is to optimise the performance and/or identify the parameters, while in the plant tuning problem the only aim is to reach the target output. It is also worth mentioning that the goal of driving the output to a desired value could be cast in terms of minimizing the norm of the error and faced as an extremum-seeking problem [15], [18], [20]. There are also interesting connections with robust optimization (see [4] for an extensive survey).

To the best of our knowledge, the approach proposed in [5], [6] is the only one that exploits the inclusion of the Jacobian in a robustly nonsingular polytope. This inclusion is a strong assumption, yet satisfied in many contexts.

In this paper, we discuss what happens if the plant is *not perfectly static*, but has a parasitic dynamics, with *unknown time constants*. This question was left open in [6], where we just argued that the problem should be faced by keeping the tuning gain small. Here we actually solve the problem and provide a suitable bound for the gain.

The contributions of this paper are the following.

- We face the tuning problem for a plant with unknown parasitic dynamics and provide a solution that relies on the exclusive knowledge of the polytope $\mathcal{M}$ in which the Jacobian is confined and of an upper bound Θ_{max} for the system time constants.
- We prove that the tuning strategy proposed in [6] based on the min-max theorem can be effectively applied, provided that the tuning gain is sufficiently small.
- We provide an upper bound for the gain that ensures closed-loop stability, based exclusively on the knowledge of $\mathcal{M}$ and Θ_{max}.
- We consider the case in which the parasitic dynamics do not affect the plant, but the actuators, and we show that a similar upper bound can be provided as well.

[a] Dipartimento di Matematica, Informatica e Fisica, Università degli Studi di Udine, 33100 Udine, Italy. blanchini@uniud.it
[b] Dipartimento di Ingegneria e Architettura, Università degli Studi di Trieste, 34127 Trieste, Italy. {fenu, fapellegrino}@units.it
[c] Delft Center for Systems and Control, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. g.giordano@tudelft.nl

978-1-5090-2873-3/17/$31.00 ©2017 IEEE

II. Motivating Example

The flow network shown in Fig. 1 was considered as an example in [6], where it was assumed that there is no buffer capacity at the nodes. Vector $y = [y_1\ y_2\ y_3\ y_4]^\top$ represents the relative output flow at the four nodes, with respect to the flow reference $\bar{r}$; the flow corresponding to each link is operated by a variable u_k, with $u = [u_1\ u_2\ u_3\ u_4\ u_5\ u_6]^\top$, where the value of u_k is given by an *unknown* function $\phi_k(u_k)$. It is only known that $\phi_k(u_k)$ are increasing for all $k = 1, \ldots, 6$. This situation is typical in channel (or pipe) networks, in which the flows are regulated by locks (or valves): the control variable is then the lock opening fraction, while the corresponding flow is not known exactly; however, it is absolutely reasonable to assume that the flow functions $\phi_k(\cdot)$ are strictly increasing. Given the flow reference $\bar{r}$, the model output in the static case (no buffer capacity) is simply

$$y = B\phi(u) - \bar{r},$$

where B is the incidence matrix of the network graph, which in the case of Fig. 1 is

$$B = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}.$$

For such a system, the results in [6] provide a robust tuner, designed according to the scheme in Fig. 2, which drives y to zero by only knowing upper and lower bounds on the derivatives of the functions $\phi_k(\cdot)$, no matter how "conservative": any bound of the form $\epsilon \leq \phi_k'(\cdot) \leq \mu$, with small ϵ and large μ, is suitable.

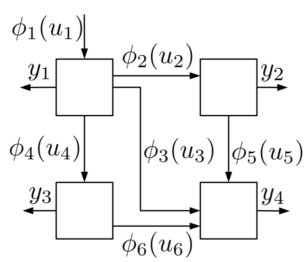

Fig. 1: The flow network problem.

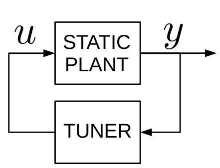

Fig. 2: The automatic tuning scheme.

The situation is completely different if the reservoirs have a capacity (with filled volume h) and an associated dynamics

$$\dot{h} = -\Lambda h + B\phi(u) - \bar{r},$$

where Λ is a diagonal matrix with positive diagonal entries. We can take the outgoing flow as the system output and assume it is proportional to the reservoirs volume:

$$y = \Lambda h.$$

Denoting by $\Theta = \Lambda^{-1}$, we get

$$\Theta \dot{y} = -y + B\phi(u) - \bar{r}.$$

Our purpose is to bring y to 0, so that $\bar{r} = B\phi(u)$. However, in this case, the straightforward application of the tuning law proposed in [6] would not give any stability guarantee. In this paper, we show how to design a tuning law that achieves our plant-tuning goal and only requires the knowledge of the polytope where the system Jacobian is known to take values and an upper bound for the entries of Θ.

III. Preliminary Results

A. The Static Case

We briefly report here the main results of [6], which considers the tuning problem for static plants only.

Problem 1: Given the static plant

$$y = g(u), \tag{1}$$

where $g : \mathbb{R}^m \to \mathbb{R}^p$, $p \leq m$, is a smooth function, assume that $g(\bar{u}) = 0$ for some *unknown* $\bar{u}$ and that the following inclusion holds:

$$G_u \doteq \left[\frac{\partial g}{\partial u}\right] \in \mathcal{M}, \tag{2}$$

where G_u is the Jacobian of g and $\mathcal{M}$ is a known polytope (or any convex and compact set) of matrices. Find a dynamic algorithm such that, as $t \to \infty$,

$$\begin{aligned} y(t) &\rightarrow 0, & (3) \\ u(t) &\rightarrow \bar{u}, & (4) \end{aligned}$$

where $\bar{u}$ solves the equation

$$0 = g(u). \tag{5}$$

$\diamond$

The only available information for tuning purposes is (2).

Definition 1: A polytope $\mathcal{M}$ is *robustly right invertible* if any matrix in $\mathcal{M}$ is right invertible. $\diamond$

We consider a control scheme of the form

$$\begin{aligned} \dot{u}(t) &= v(t), & (6) \\ v(t) &= \Phi(y(t)), & (7) \\ y(t) &= \text{measured output}, & (8) \\ v(t) \in \mathcal{V} &= \{v : \|v\| \leq \xi(y)\}, & (9) \end{aligned}$$

where $\xi(y) > 0$ is a continuous, nonnegative and non-decreasing function, while $\|\cdot\|$ is any norm.

Theorem 1: If the polytope $\mathcal{M}$ in (2) is robustly right invertible, Problem 1 can be solved by a control scheme (6)-(8), with v bounded as in (9). $\square$

The control is computed on-line by considering the convex optimization problem

$$\hat{M}(y) = \arg\min_{M \in \mathcal{M}} \|y^\top M\|_*, \tag{10}$$

where $\|\cdot\|_*$ is the dual norm of $\|\cdot\|$. Then

$$v = \Phi(y) \doteq -\frac{M^\top y}{\|M^\top y\|}\xi(y), \quad y \neq 0. \tag{11}$$

For $y = 0$, we assume $v = 0$. Note that, if we take $\xi(y) = \xi\|y\|$, we have continuity at 0 and the control is bounded as

$$\|v^*(y)\| \leq \xi\|y\|. \tag{12}$$

Henceforth, we let $\|\cdot\|$ denote the Euclidean norm.

B. Some Technical Lemmas

We introduce here some technical lemmas that will be important in the sequel.

Lemma 1: Consider the control $v = v^*(y)$ as in (11), which satisfies (12) for the static problem, and the Lyapunov-like function $V(y) = y^\top y/2$. Then

$$\dot{V}(y) = y^\top G_u v^* \leq -\gamma\|y\|^2, \tag{13}$$

with

$$\gamma = \nu\xi, \tag{14}$$

where ξ is the scalar in (12) and $\nu > 0$ is the smallest possible value of the smallest singular value $\underline{\sigma}(M)$ of a matrix M in the polytope:

$$\nu \doteq \min_{M\in\mathcal{M}} \underline{\sigma}(M). \tag{15}$$

□

Proof: Denoting by $\hat{M}^\top y$ the minimum-norm element in the convex set $\{r = M^\top y, \quad M \in \mathcal{M}\}$, we have

$$\min_{M\in\mathcal{M}} y^\top M\hat{M}^\top y = y^\top \hat{M}\hat{M}^\top y.$$

Hence, being $G_u \in \mathcal{M}$,

$$\begin{aligned} -\dot{V} &= -y^\top G_u v^*(y) \geq \min_{M\in\mathcal{M}} y^\top M\frac{\hat{M}^\top y}{\|\hat{M}^\top y\|}\xi\|y\| \\ &= y^\top \hat{M}\frac{\hat{M}^\top y}{\|\hat{M}^\top y\|}\xi\|y\| = \xi\|y\|\|\hat{M}^\top y\| \geq \nu\xi\|y\|^2. \end{aligned}$$

■

To face the actuator dynamics, we need a lemma from [6].

Lemma 2: Define variable z as

$$z = u - w. \tag{16}$$

Then,

$$g(u) = g(w) + \tilde{G}_{u,z}z \tag{17}$$

for some $G_{u,z} \in \mathcal{M}$. □

Proof: Consider the formula [14, p. 156, Exercise 3.9]

$$g(u) - g(w) = \left[\int_0^1 \frac{\partial f}{\partial u}(\sigma z)d\sigma\right] z \doteq \tilde{G}_{u,w}z. \tag{18}$$

Since $\frac{\partial f}{\partial u} \in \mathcal{M}$, and the integral is the average, the integral belongs to $\mathcal{M}$ in view of convexity. ■

Remark 1: The previous lemma ensures uniqueness of the solution $\bar{u}$, if the Jacobian is square and robustly nonsingular. Indeed, for two different solutions $\bar{u}_1$ and $\bar{u}_2$ we would have

$$g(\bar{u}_2) - g(\bar{u}_1) = G_{u,z}(\bar{u}_1 - \bar{u}_2) = 0$$

for some (nonsingular) $G_{u,z} \in \mathcal{M}$, which is impossible.

IV. Non-Static Plants

In several cases, a plant that is "practically static" actually has a parasitic dynamics, which, for slow variations of the input, does not affect the problem solution. Unfortunately, if the tuning action is aggressive, the dynamics can come into play and needs to be taken into account. Hence, to prevent the risk of closed-loop instability, the control must be slow enough. To deal with this new problem, we consider the tuning law proposed for static plants and show that it can be used for dynamic plants as well, if γ is taken small enough.

Assume that the plant has dynamics

$$\Theta\dot{y} = -y + g(u), \tag{19}$$

where $\Theta = \mathrm{diag}\{\theta_1, \ldots, \theta_m\}$ is an unknown matrix with time-constant diagonal entries.

Assumption 1: The only available information about the diagonal matrix Θ is the upper bound

$$\max_{i=1,\ldots,m}\{\theta_i\} \leq \Theta_{max}. \tag{20}$$

◇

To tackle the problem with parasitic dynamics, the main idea comes from the singular perturbation method [14]. Continuity of $G_u \in \mathcal{M}$ guarantees

$$\|G_u\| \leq \mu \tag{21}$$

for some positive constant μ (we can simply take the largest singular value of all $M \in \mathcal{M}$). Then, the next theorem shows how to limit the control action $\|v\| \leq \xi$, so as to ensure stability, exclusively based on the upper bound (20).

Theorem 2: Given system (19), consider the control $v = v^*(y)$ as in (11) and assume it satisfies the bound (12) for the static problem, where $\xi > 0$ is a decision variable. Let ν be defined as in (15) and μ as in (21). Then, the closed-loop dynamic model is stable if

$$\xi < \frac{4\nu}{\mu^2(1+\Theta_{max})^2}. \tag{22}$$

Moreover, $y(t) \to 0$ and $u(t) \to \bar{u}$. □

Proof: Without restriction, we assume that the desired equilibrium is $\bar{y} = 0$ and is associated with the control $\bar{u} = 0$. Indeed, if $\bar{y} = g(\bar{u})$, we write

$$\delta y = y - \bar{y} = g(\delta u + \bar{u}) - g(\bar{u}) = \delta G(\bar{u}).$$

It is worth stressing that the adopted translation of the variables $u \to \delta u$ and $y \to \delta y$ does not affect the Jacobian δG, nor the integral action:

$$\dot{\delta u} = \dot{u} = v.$$

Obviously, the control is based on the knowledge of the target output $\bar{y}$ only, and not of the corresponding input $\bar{u}$. The overall dynamics, in the dynamical plant case, is

$$\begin{aligned} \dot{u} &= v, \\ \Theta\dot{y} &= -y + g(u). \end{aligned}$$

Let us introduce the variable

$$z = y - g(u)$$

and note that $\Theta\dot{y} = -z$. Then

$$\Theta\dot{z} = \Theta\dot{y} - \Theta\frac{d}{dt}g(u) = -z - \Theta\frac{\partial g(u)}{\partial u}\dot{u}.$$

Hence,

$$\begin{aligned} \dot{u} &= v, \\ \Theta\dot{z} &= -z - \Theta G_u v. \end{aligned}$$

Let us introduce the candidate Lyapunov function

$$W(u,z) = \frac{1}{2}\|g(u)\|^2 + \frac{1}{2}z^\top \Theta z,$$

which is positive definite, as long as $g(u) = 0$ if and only if $u = 0$. Since $g(u) = y - z$, the Lyapunov derivative is

$$\begin{aligned} \dot{W} &= g(u)^\top \frac{\partial g(u)}{\partial u}\dot{u} + z^\top \Theta \dot{z} \\ &= g(u)^\top G_u v + z^\top[-z - \Theta G_u v] \\ &= [y-z]^\top G_u v - \|z\|^2 - z^\top \Theta G_u v \\ &= y^\top G_u v - \|z\|^2 - z^\top [I + \Theta] G_u v. \end{aligned}$$

If we apply the control $v = v^*(y)$, in view of (12) and (21), we have that $\|G_u v\| \leq \mu\xi\|y\|$. Moreover, considering that $\gamma = \nu\xi$ in view of (14),

$$\begin{aligned} \dot{W} &\leq -\gamma\|y\|^2 - \|z\|^2 + (1+\Theta_{max})\mu\xi\|z\|\|y\| = \\ &\begin{bmatrix}\|y\| & \|z\|\end{bmatrix} \begin{bmatrix} -\nu\xi & (1+\Theta_{max})\frac{\mu\xi}{2} \\ (1+\Theta_{max})\frac{\mu\xi}{2} & -1 \end{bmatrix} \begin{bmatrix}\|y\| \\ \|z\|\end{bmatrix}. \end{aligned}$$

The matrix in the above expression is negative definite for $\xi > 0$ small enough. Indeed, since its $(1,1)$ entry is negative, we just need to make sure that its determinant is positive: $\nu\xi - \xi^2\mu^2(1+\Theta_{max})^2/4 > 0$, which corresponds to the condition in (22). The proof is concluded by noticing that

$$\begin{bmatrix}\|y\| \\ \|z\|\end{bmatrix} = 0$$

if and only if $y = 0$ and $u = 0$. Indeed, $z = -g(u)$ if $y = 0$, and $z = -g(u) = 0$ if also $u = 0$, since $g(0) = 0$. ■

V. Actuator Dynamics

We can also consider parasitic dynamics affecting the actuators:

$$\begin{aligned} y &= g(w) &(23) \\ \Theta\dot{w} &= -w + u &(24) \\ \dot{u} &= v &(25) \\ v &= \Phi(y) &(26) \end{aligned}$$

We consider the same strategy $v^* = \Phi(y)$ adopted for static plants, which satisfies the bound

$$\|v^*(y)\| \leq \xi\|y\|. \qquad (27)$$

With this control choice, the Lyapunov function $V(y) = y^\top y/2$ satisfies

$$\dot{V}(y) = y^\top G_u v^* \leq -\nu\xi\|y\|^2 \qquad (28)$$

for any $G_u \in \mathcal{M}$.

Also, for $\tilde{G}_{u,z} \in \mathcal{M}$ and $G_w \in \mathcal{M}$, we have

$$\|\tilde{G}_{u,z}^\top G_w\| \leq \mu^2, \qquad (29)$$

where μ is defined in (21).

The next theorem holds.

Theorem 3: Given system (23)–(26), the control $v = v^*(y)$ as in (11), bounded as in (27) and satisfying the condition (28), stabilizes the dynamic plant and ensures $y(t) \to 0$ and $u(t) \to \bar{u}$, provided that

$$\xi < \frac{4\nu}{(\mu^2 + \Theta_{max})^2}. \qquad (30)$$

Proof: Assuming $\bar{u} = 0$ without restriction, we define

$$z = u - w$$

and we consider the Lyapunov-like function

$$W(u,z) = \frac{1}{2}\|g(u)\|^2 + \frac{1}{2}z^\top \Theta z,$$

which is not the same function we adopted before, because z is defined differently. Function $g(u)$ is evaluated in u, not in w. Since $z = \Theta\dot{w}$ in view of (24) and $g(u) = g(w) + \tilde{G}_{u,z}z$ in view of (17), cf. Lemma 2, the derivative is

$$\begin{aligned} \dot{W} = \;& g(u)^\top \frac{\partial g(u)}{\partial u}\dot{u} + z^\top \Theta\dot{z} = g(u)^\top G_u v + z^\top \Theta[\dot{u} - \dot{w}] \\ =\;& g(w)^\top G_u v + z^\top \tilde{G}_{u,z}^\top G_u v + z^\top \Theta v - z^\top z \\ =\;& y^\top G_u v + z^\top \tilde{G}_{u,z}^\top G_u v + z^\top \Theta v - \|z\|^2 \\ \leq\;& -\xi\nu\|y\|^2 + (\mu^2\xi + \Theta_{max}\xi)\|z\|\|y\| - \|z\|^2 < 0 \end{aligned}$$

for $y \neq 0$ and $z \neq 0$ if (30) holds. Then $(z(t), y(t)) \to 0$. ■

VI. Examples

A. The Flow Problem

Reconsider the flow problem discussed in Section II, associated with the network shown in Fig. 1 and with equations

$$y = B\phi(u) - \bar{r},$$

where $\phi(u)$ is a vector of strictly increasing smooth functions $\phi(u) = [\phi_1(u_1)\ \phi_2(u_2) \ldots, \phi_6(u_6)]^\top$. The Jacobian is

$$B\text{diag}\{\phi_1'(u_1), \phi_2'(u_2), \ldots, \phi_6'(u_6)\} = BD$$

where D is a diagonal matrix with positive diagonal entries, hence BD has full rank. The bounds on the derivatives are

$$1 \leq D_i \leq 4, \quad i = 1, \ldots, 6.$$

We assume $\Theta_{max} = 1s^{-1}$. The above are the *only information we need* for tuning purposes. The corresponding maximum gain value, in view of condition (22), is

$$\xi_{max} = 0.0571.$$

The reference is $\bar{r} = [1\ 1 \ldots 1]^\top$. The parameters adopted for the simulations, *which are not available for design*, are

$$\Theta = \text{diag}\{1, 1, 1, 1\}$$

and the functions of the form

$$\alpha_i u_i + \beta_i \text{atan}(u_i)$$

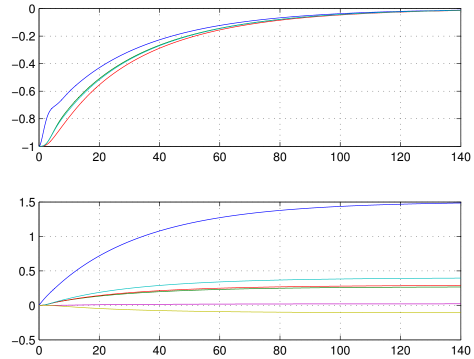

Fig. 3: The evolution of y (top) and u (bottom) with $\xi = \xi_{max} = 0.0571$.

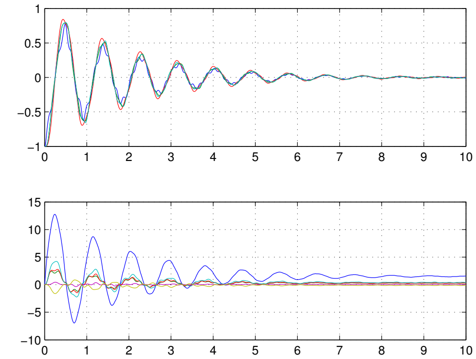

Fig. 5: The evolution of y (top) and u (bottom) with $\xi = 100$.

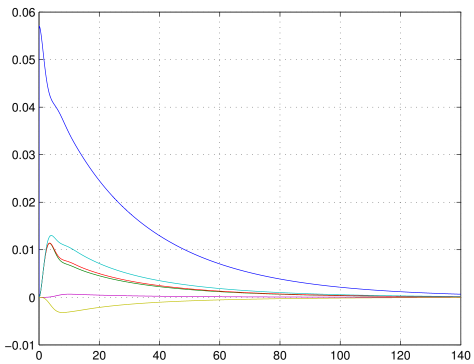

Fig. 4: The evolution of v with $\xi = \xi_{max} = 0.0571$.

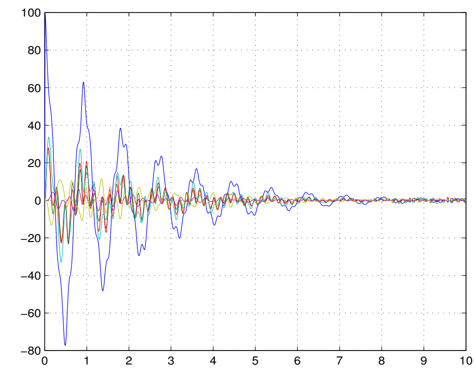

Fig. 6: The evolution of v with $\xi = 100$.

with the coefficients α_i and β_i listed below

	1	2	3	4	5	6
α_i	2	2	2	1	2	1
β_i	1	2	2	1	1	1

If we choose $\xi = \xi_{max} = 0.0571$, we get the smooth and converging behaviour reported in Figures 3 and 4. In the absence of dynamics, any gain would be suitable. However, in the presence of dynamics, very nasty oscillations arise if the gain is too large. For instance, with $\xi = 100$, we get the transient reported in Figures 5 and 6.

Remark 2: Although in principle convergence is numerically achieved, the transient produces very large oscillations at the beginning, which are in practice unacceptable. During these oscillations, the flows reach values that are completely out of range: they are two orders of magnitude larger than the steady-state values, which are equal to 1.

B. Slow Actuators

Consider the planar cable robot represented in Fig. 7. Two motors in positions A_1 and A_2 can regulate the length of two cables with pulleys of unknown diameter. The motors are actuated by providing a reference angle, with a time constant due to a low-pass filter. The actuator equations are

$$\theta_k \dot{l}_k = (-l_k + u), \quad k = 1, 2,$$

where l_k is the length of the released cable, which is proportional (with a possibly uncertain constant) to the pulley angles. The two pulleys are at an unknown distance d (and not necessarily at the same height). The motor time constants are unknown but bounded by $\Theta_{max} = 1s^{-1}$.

The target is to drive the robot end-effector to a certain position (x_T, z_T). The end-effector position $[x, z]^\top$ is measured by a camera and is a function of the length $[l_1, l_2]^\top$. The Jacobian has the form

$$M = \begin{bmatrix} m_{11} & -m_{12} \\ -m_{21} & -m_{22} \end{bmatrix}$$

and reasonable bounds $\epsilon \leq m_{ij} \leq \delta$ can be provided, valid in a certain operating region. *No other information is available*.

For simulation purposes we take $\theta_1 = \theta_2 = 1s^{-1}$, the pulleys at the same level, at a distance of $d = 10m$, $\epsilon = 0.2$

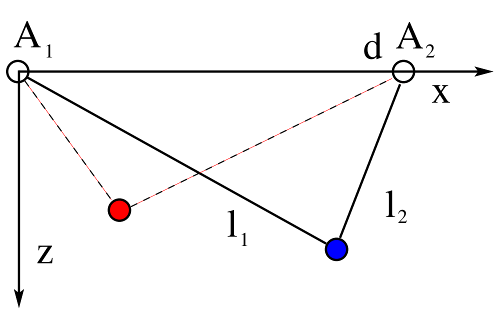

Fig. 7: The cable robot.

and $\delta = 1$. The adopted functions are

$$x = \frac{d^2 + l_1^2 - l_2^2}{2d} \text{ and } y = \sqrt{l_1^2 - \left[\frac{d^2 + l_1^2 - l_2^2}{2d}\right]^2}$$

The target position is $(x_T, z_T) = (4, -6)$ and the initial position is $(x_0, z_0) = (7, -9)$.

In Fig. 8 we show the transient with $\xi = 100$. Again, the system converges but has unacceptable oscillations, which would completely invalidate the scheme. In Fig. 9 we show the transient with $\xi = \xi_{max} = 0.1257$, computed according to our results: as expected, the transient is quite smooth.

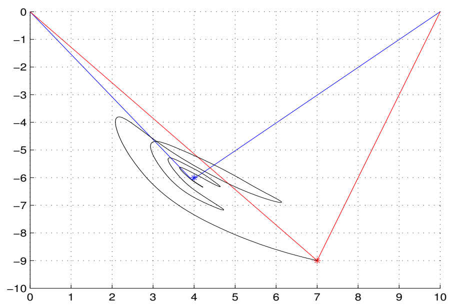

Fig. 8: Transient with $\xi = 100$

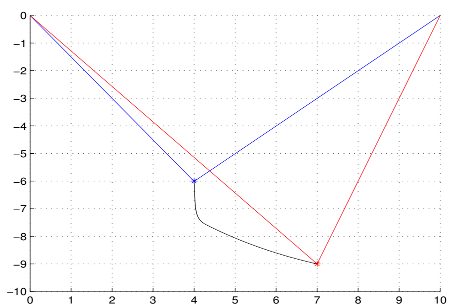

Fig. 9: Transient with $\xi = \xi_{max} = 0.1257$

The overshooting transient is faster ($15s$) than the smooth one ($150s$). This happens because the choice of ξ_{max} is conservative, and can be a problem if the transient performance, which we do not consider at all in the paper, is crucial. To have a fast and suitably damped transient, some kind of identification of the system parameters would be necessary.

VII. Concluding Discussion

Plant tuning is often a frustrating operation because, due to the lack of reliable models, it requires trial-and-error procedures. As shown in [5], [6], under suitable assumptions on the Jacobian of the unknown plant model, a static plant can be tuned by means of an automatic procedure.

In this paper, we have addressed a question left open in [6]: what happens if the plant is not perfectly static, but has unknown parasitic dynamics? We have shown that the technique presented in [6] can be applied without variations, provided that the tuning gain is sufficiently small. Upper bounds for a suitable tuning gain have been provided, based exclusively on the upper bound of the unknown time constants of the parasitic dynamics, for both the cases of parasitic dynamics affecting the plant and affecting the actuators.

These bounds express a compromise between the performances loss due to either a low gain, which means a slow convergence, or a high gain, which can compromise the mechanism, causing instability or unacceptable oscillations. The proposed bounds may be conservative, leading to small gain values. In practice this is a problem which can be adaptively solved on–line by adjusting the gain based on some estimation of the current value of the Jacobian.

References

[1] H.-S. Ahn, Y. Q. Chen, and K. L. Moore, “Iterative learning control: Brief survey and categorization”, *IEEE Transactions on Systems, Man and Cybernetics, Part C: Applications and Reviews*, vol. 37, pp. 1099-1121, 2007.

[2] K. J. Åström, “Theory and applications of adaptive control – A survey”, *Automatica*, vol. 19, no. 5, pp. 471–486, 1983.

[3] F. Blanchini, “The gain scheduling and the robust state feedback stabilization problems”, *IEEE Transactions on Automatic Control*, vol. 45, no. 11, pp. 2061–2070, 2000.

[4] H. G. Beyer and B. Sendhoff, “Robust optimization – A comprehensive survey”, *Computer Methods in Applied Mechanics and Engineering*, vol. 196, no. 33–34, pp. 3190–3218, 2007.

[5] F. Blanchini, G. Fenu, G. Giordano, F. A. Pellegrino, “Plant tuning: a robust Lyapunov approach”, *Proceedings of the 54th IEEE Conference on Decision and Control*, Osaka, December 15-18, 2015.

[6] F. Blanchini, G. Fenu, G. Giordano, F. A. Pellegrino, “Model-free plant tuning”, *IEEE Transactions on Automatic Control*, vol. 62, no. 6, pp. 2623–2634, 2017.

[7] F. Blanchini, G. Fenu, G. Giordano, F. A. Pellegrino, “Discrete-time plant tuning without a model”, *Proceedings of the 20th IFAC World Congress*, 2017.

[8] F. Blanchini and R. Pesenti, “Min-max control of uncertain multi-inventory systems with multiplicative uncertainties”, *IEEE Transactions on Automatic Control*, vol. 46, no. 6, pp. 955–959, 2001.

[9] D. A. Bristow, M. Tharayil, and A. G. Alleyne, “A survey of iterative learning control,” *IEEE Control Systems*, vol. 26, no. 3, pp. 96–114, June 2006.

[10] A. Fradkov, “A scheme of speed gradient and its application in problems of adaptive control”, *Autom. and Rem. Contr.*, vol. 40, no. 9, pp. 1333–1342, 1980.

[11] R. A. Freeman and P. V. Kokotović, *Robust Nonlinear Control Design. State-space and Lyapunov Techniques*, Systems and Control: Foundations and Applications. Birkhäuser Inc., Boston, MA, 1996.

[12] S. Gutman and G. Leitmann, “Stabilizing control for linear systems with bounded parameter and input uncertainty”, *Optimization Techniques Modeling and Optimization in the Service of Man* Part 2, Lecture Notes in Computer Science, pp. 729-755, 1976.

[13] S. Gutman, “Uncertain dynamical systems — a Lyapunov min-max approach”, *IEEE Transactions on Automatic Control*, vol. 24, no. 3, pp. 437–443, 1979.

[14] H. Khalil. *Nonlinear systems*. Prentice Hall, Upper Saddle River, NJ, 1996.

[15] S. Z. Khong, D. Nešić, Y. Tan, C. Manzie, “Unified frameworks for sampled-data extremum seeking control: global optimisation and multi-unit systems”, *Automatica*, vol. 49, no. 9, pp. 2720–2733, 2013.

[16] D. G. Luenberger, *Optimization by vector space methods*. John Wiley & Sons Inc., New York, 1969.

[17] A. M. Meılakhs, “Design of stable control systems subject to parametric perturbation”, *Autom. Rem. Control*, vol. 39, no. 10, pp. 1409–1418, 1979.

[18] D. Nešić, A. Mohammadi, C. Manzie, “A framework for extremum seeking control of systems with parameter uncertainties”, *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 435–448, 2013.

[19] Z. Qu., *Control of Nonlinear Uncertain Systems*, Wiley, New York, 1998.

[20] Y. Tan, D. Nešić and I. Mareels, “On non-local stability properties of extremum seeking control”, *Automatica*, vol. 42, no. 6, pp. 889–903, 2006.