Chapter 15
On the LPV Control Design and Its Applications to Some Classes of Dynamical Systems

Franco Blanchini, Daniele Casagrande, Giulia Giordano and Stefano Miani

Abstract In this chapter, a control design approach based on linear parameter-varying (LPV) systems, which can be exploited to solve several problems typically encountered in control engineering, is presented. By means of recent techniques based on Youla–Kucera parametrization, it is shown how it is possible not only to design and optimize stabilizing controllers, but also to exploit the structure of the Youla–Kucera parametrized controller to face and solve side problems, including: (a) dealing with nonlinearities; (b) taking into account control input constraints; (c) performing controller commutation or online adaptation, e.g., in the presence of faults; and (d) dealing with delays in the system. The control scheme is observer-based, namely a prestabilizing observer-based precompensator is applied. Consequently, a Youla–Kucera parameter is applied to produce a supplementary input ignition, which is a function of the residual value (the difference between the output and the estimated output). Based on the fact that any stable operator which maps the residual to the supplementary input preserves stability, several additional features can be added to the compensator, without compromising the loop stability.

Keywords Linear parameter-varying (LPV) systems · Youla–Kucera parametrization · Actuator and sensor faults · Time-delay systems · Control of saturated systems

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15.1 Introduction

Linear parameter-varying (LPV) systems constitute a class of linear time-varying systems which lay in between uncertain systems and linear systems and allow for an elegant and effective description of many dynamic systems for which the knowledge of the current configuration is known pointwise in time, [29]. Such systems have been studied since the 1990s (see [13, 15, 20, 22, 24, 26, 28, 33]) and can be thought of as linear time-invariant plants with time-varying, uncertain but pointwise-in-time known, parameters. The parametric structure may be intrinsic in the physical system or may appear in the model, resulting, e.g., from the linearization of nonlinear systems in different operating points, [1, 30], or from the adoption of different controllers, each acting according to the designer-specified switching role, [5, 7, 14]. This last point of view, say the analysis of LPV systems as the result of the combination of a linear (possibly time-varying) system along with a scheduled controller, has provided (i) a full comprehension of phenomena occurring during the system commutation, and (ii) a full and exhaustive characterization of the stabilizing controllers that can be adopted for this class of systems.

In this chapter, we will review the newly proposed results in this area and we will provide several examples of systems for which the provided theory guarantees an effective solution. These examples, aimed at bridging different fields under the common denominator of LPV systems, span from the case of control in the presence of actuator and sensor faults, for which effective results have been provided in [32], to the case of control of time-delay systems, [6, 18, 21], and to the case of control of saturated systems, [12].

15.2 LPV Systems: Definition and Main Results

The class of LPV systems is described by the n-dimensional system with m inputs and p outputs

$$\begin{aligned} \sigma(x(t)) &= A(w(t))x(t) + B(w(t))u(t), \\ y(t) &= C(w(t))x(t), \end{aligned} \tag{15.1}$$

where $w(\cdot)$ is a function taking values in an assigned compact set $\mathcal{W}$ and $\sigma(x(t))$ represents the differential operator in the continuous-time case and the single step shift in the discrete-time case. The current value of $w(t) \in \mathcal{W}$ is known and available for control purposes, whereas its future evolution is not.

Example 15.1 Consider a simple pendulum of length l and mass M. Its dynamics are

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= -\frac{g}{l}\sin(x_1(t)) - \frac{b}{Ml^2}x_2(t) + \frac{1}{Ml^2}u(t), \\ y(t) &= x_1(t), \end{aligned}$$

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where g is the gravity acceleration, b is the constant viscous coefficient, x_1 represents the angle, x_2 the angular velocity and u the control input. Since the current value of $x_1(t) = y(t)$ is known, the system can be "embedded" in an LPV system (precisely, this particular case of LPV is also known in the literature as *quasi-LPV*, see [23] and the references therein), e.g., by rewriting the nonlinear term $\sin(x_1(t))$ as $\frac{\sin(x_1(t))}{x_1(t)} x_1(t)$ and by defining $w(t) = \frac{\sin(x_1(t))}{x_1(t)}$, with $\mathcal{W} = [-0.2172\ 1]$. By means of this parameter, the dynamics of the systems can be written in the form (15.1) with

$$A(w(t)) = \begin{bmatrix} 0 & 1 \\ -\frac{gw(t)}{l} & -\frac{b}{Ml^2} \end{bmatrix}, \quad B(w(t)) = \begin{bmatrix} 0 \\ \frac{1}{Ml^2} \end{bmatrix}, \quad C(w(t)) = [1 \quad 0].$$

In this particular case, matrices B and C do not actually depend on w. ◊

Other significant examples of LPV systems will be reported along the chapter.

Definition 15.1 System (15.1) is *LPV stable* if the zero equilibrium is asymptotically stable for any function $w : [0, +\infty) \to \mathcal{W}$.

It is a rather established fact that stability of $A(w)$ for every constant value of $w \in \mathcal{W}$ is just a necessary condition for the stability of (15.1) in the sense of Definition 15.1.

The asymptotic stability of the zero equilibrium is equivalent to the existence of a Lyapunov function.

The peculiarity of LPV systems, as far as control design is concerned, lies in the fact that the future evolution of the time-varying parameter $w(t)$ is unknown, but its current value is known. This characteristic allows us to derive nonconservative conditions for the existence of an LPV stabilizing regulator based on linear matrix inequalities (LMIs), [2, 3, 9, 20, 25], as per the following result proved in [3, 5].

Theorem 15.1 *The LPV system (15.1) is (quadratically) stabilizable via a n-dimensional observer—based LPV regulator if and only if there exist two symmetric positive—definite matrices P and Q, both in* $\mathbb{R}^{n\times n}$*, and two matrices* $U(w) \in \mathbb{R}^{m\times n}$ *and* $Y(w) \in \mathbb{R}^{n\times p}$ *such that the following set of LMIs (in the continuous-time and in the discrete-time case, respectively) is satisfied for every* $w \in \mathcal{W}$.

- *Continuous-time:*

$$PA(w)^\top + A(w)P + B(w)U(w) + U(w)^\top B(w)^\top \prec 0, \tag{15.2}$$

$$A(w)^\top Q + QA(w) + Y(w)C(w) + C(w)^\top Y(w)^\top \prec 0. \tag{15.3}$$

- *Discrete-time:*

$$\begin{bmatrix} P & (A(w)P + B(w)U(w))^\top \\ A(w)P + B(w)U(w) & P \end{bmatrix} \succ 0, \tag{15.4}$$

$$\begin{bmatrix} Q & (QA(w) + Y(w)C(w))^\top \\ QA(w) + Y(w)C(w) & Q \end{bmatrix} \succ 0. \tag{15.5}$$

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If the above conditions are satisfied for every $w \in \mathcal{W}$, then the observer—based control law

$$\begin{aligned}\sigma(\hat{x}(t)) = &[A(w(t)) + L(w(t))C(w(t)) + B(w(t))J(w(t))]\,\hat{x}(t)\\ &-L(w(t))y(t) + B(w(t))v(t),\\ u(t) = &J(w(t))\hat{x}(t) + v(t),\end{aligned} \tag{15.6}$$

with $v(t) = 0$,[1] *is stabilizing. The observer and estimated state gains are*

$$J(w(t)) = U(w(t))P^{-1} \tag{15.7}$$

and

$$L(w(t)) = Q^{-1}Y(w(t)). \tag{15.8}$$

Remark 15.1 The conditions reported in the previous theorem guarantee the existence of a Luenberger observer-based controller of the same dimension of the plant. The interested reader is referred to [5] for necessary and sufficient conditions for the existence of a **linear** extended observer when the LMI conditions just reported fail.

It is worth stressing that the stabilizability conditions might correspond to an infinite number of LMIs. However, there are important cases in which such a set of LMIs reduces to a finite number, thus the solution is easily computable by means of standard software tools. Two interesting cases are the following.

1. Systems where the input and output matrices are constant, while $A(w(t))$ is polytopic [1]: $A(w(t)) = \sum_{i=1}^{r} A_i w_i(t)$, with $w_i(t) \geq 0\ \forall i$ and $\sum_{i=1}^{r} w_i(t) = 1$.
2. Systems belonging to the class of switching systems, [7, 16], described by

$$\begin{aligned}\sigma(x(t)) &= A_{i(t)}x(t) + B_{i(t)}u(t),\\ y(t) &= C_{i(t)}x(t),\end{aligned}$$

where $i(t) \in \{1, \ldots, r\}$ is a switching signal.

In both cases, the set of $r + r$ LMIs to be solved is the following (the continuous-time case LMIs only are reported):

$$PA_i^\top + A_iP + B_iU_i + U_i^\top B_i^\top \prec 0, \qquad i = 1, \ldots, r \tag{15.9}$$

$$A_i^\top Q + QA_i + Y_iC_i + C_i^\top Y_i^\top \prec 0, \qquad i = 1, \ldots, r \tag{15.10}$$

with the understanding that $B_i = B$ for all i and $C_i = C$ for all i if the input and output matrices are constant.

[1] The signal $v(t)$ is introduced in (15.6) to avoid duplicating the observer-based stabilizing regulator equations and will be used later, when the Youla–Kucera parameter will come into play.

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In case 1, the stabilizing observer—based controller is

$$\begin{aligned}\sigma(\hat{x}(t)) &= \sum_{i=1}^{r}\left[A_i + L_iC + BJ_i\right] w_i(t)\hat{x}(t) - \sum_{i=1}^{r} L_i w_i(t) y(t) + Bv(t),\\ u(t) &= \sum_{i=1}^{r} J_i w_i(t)\hat{x}(t) + v(t),\end{aligned} \tag{15.11}$$

whereas in case 2 it is

$$\begin{aligned}\sigma(\hat{x}(t)) &= \left(A_{i(t)} + L_{i(t)}C_{i(t)} + B_{i(t)}J_{i(t)}\right)\hat{x}(t) - L_{i(t)}y(t) + B_{i(t)}v(t),\\ u(t) &= J_{i(t)}\hat{x}(t) + v(t)\end{aligned} \tag{15.12}$$

and, again, $J_i = U_iP^{-1}$, $L_i = Q^{-1}Y_i$, while $v(t) = 0$ (see Footnote 1).

The signal $v(t)$ appearing in Theorem 15.1, and so far set to 0, can be successfully employed to parametrize all the linear stabilizing compensators via Youla–Kucera parametrization. Moreover, if it is generated as the output of an LPV stable operator whose input is the estimation error, then the overall system remains stable, as per the next result.

Theorem 15.2 *Assume the stabilizability conditions in Theorem 15.1 are satisfied. Let*

$$o(t) = C(w(t))\hat{x}(t) - y(t) \tag{15.13}$$

and consider any globally asymptotically stable operator mapping $o(t)$ into $v(t)$, i.e.,

$$\begin{aligned}\sigma(z(t)) &= g(z(t), o(t), t),\\ v(t) &= h(z(t), o(t), t).\end{aligned} \tag{15.14}$$

Then the observer—based regulator (15.6), with $v(\cdot)$ defined by (15.14) and (15.13), is stabilizing.

Under the additional assumption of the existence of a single quadratic Lyapunov function for the closed-loop system, the converse is also true. Consider the closed-loop system obtained from (15.1) when the stabilizing controller

$$\begin{aligned}\sigma(q(t)) &= f(q(t), y(t), t),\\ u(t) &= k(q(t), y(t), t)\end{aligned} \tag{15.15}$$

is adopted and assume that such system admits a single quadratic Lyapunov function. Then the stabilizing controller (15.15) can be parametrized as in (15.6) for a proper stable operator (15.13), (15.14), which is known as the Youla–Kucera *parameter.*

The constructive proof of this theorem is reported in the Appendix, along with the procedure to compute the Youla–Kucera parameter. Here it is worth stressing that the parameterization of all the stabilizing controllers, depicted in Fig. 15.1, is exactly the classical Youla–Kucera parameterization.

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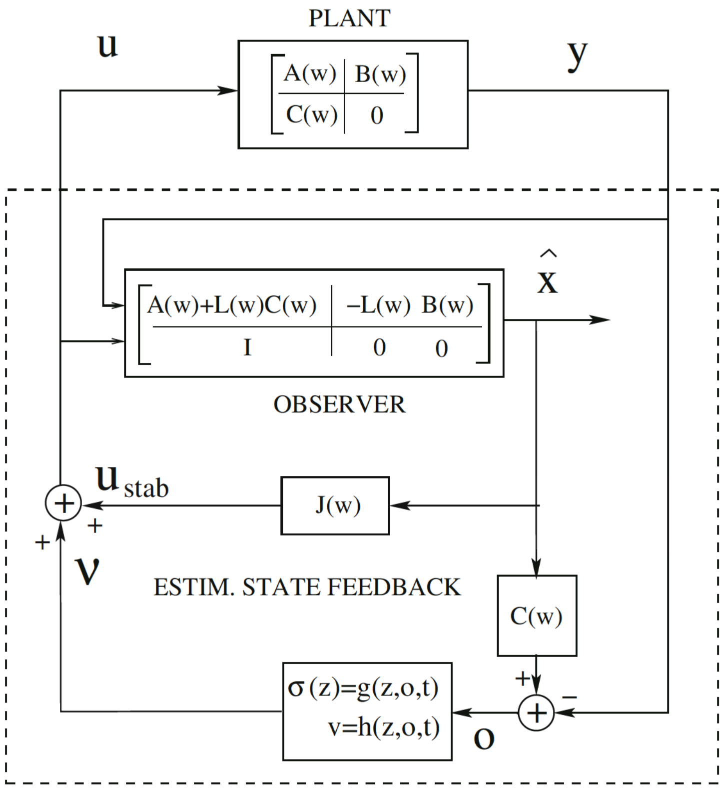

Fig. 15.1 Youla–Kucera parameterization

Apart from the case in which the stabilizing operator is itself LPV, the determination and realization of the Youla–Kucera parameter are far from being simple and will not be investigated here (the interested reader is referred to [4] and the references therein). We rather stress, once again, that the freedom in the choice of the stable operator can be successfully exploited to cope with different problems, as will be seen in the next section.

15.3 Exploiting the Youla–Kucera Parameter Choice

In this section, some applications of the proposed results will be presented, to show how it is possible to take advantage of the freedom in the choice of the Youla–Kucera parameter, so as to deal with several side problems.

15.3.1 Online Controller Adaptation Induced by Faults

In the recent literature, several LPV fault-tolerant control schemes have been analyzed (see, for instance, [27, 31]). Here, the case of real over-actuated control systems with multiple sensing channels is considered. The input and output channel redundancy is supposed to be introduced for security reasons (think, as an example, of the flap control of an airplane).

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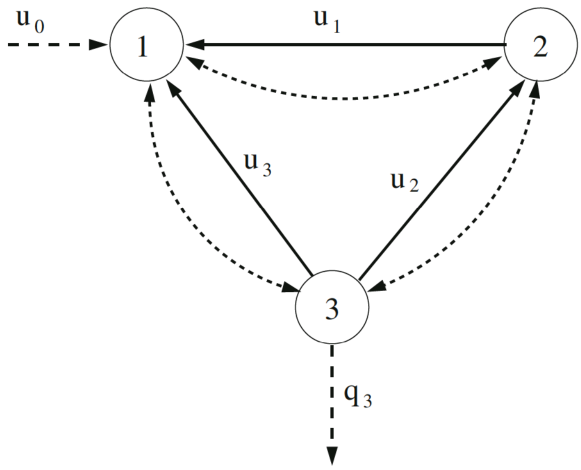

Fig. 15.2 The system of three tanks considered in Sect. 15.3.1. *Dashed lines* represent normal flows while *solid lines* represent additional flows

In such systems, rather than designing a single **robust** controller to cope with every fault scenario, it is natural to design different controllers for every possible fault configuration, so as to fully exploit the available sensors and actuators in each configuration and then commute among the different controllers when a fault is sensed. Unfortunately, since the commutations can be assumed to be random, the switching between stabilizing controllers can result in an unstable behavior. To overcome this limit, it is sufficient to verify the LMI conditions in Theorem 15.1, design the observer-based controller and then realize the obtained controllers via the Youla–Kucera parametrization.

In this example, to keep the exposition simple, only the determination of the observer—based controllers will be dealt with and the Youla–Kucera parameters will be set to zero.

The dynamics of the system we consider are

$$\begin{aligned} \dot{x}(t) &= A_{w(t)}x(t) + B_{w(t)}u(t), \\ y(t) &= C_{w(t)}x(t), \end{aligned}$$

where the integer parameter $w(t)$ is associated with the wth fault scenario. To illustrate the idea, consider the simple system formed by three connected tanks as in Fig. 15.2, in which a natural flow occurs between the different tanks, proportional to their relative levels.[2] The flow from tank k to tank h is $q_{kh}(t) = \alpha(x_k(t) - x_h(t))$, with $\alpha > 0$. Moreover, tank 3 has a discharge channel, whose flow is proportional to its level, $q_3(t) = \beta x_3(t)$, and tank 1 is fed by u_0. An additional flow between the tanks can be forced by three connecting valves (one for each pair of tanks). In the

[2]Levels and flows have to be intended as the deviation from the steady–state values.

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nominal condition, the three sensors measuring the levels work, as well as the three valves controlling the additional flow. In the present example, it is assumed that the system is always working in some faulty condition corresponding to one sensing channel and one valve working. Thus the parameter $w(t)$ can be represented as a pair (i, j) belonging to the set $\mathcal{W} = \{1, 2, 3\}^2$; the corresponding LPV system is

$$\dot{x} = Ax + B_{i(t)}u,$$
$$y = C_{j(t)}x,$$

where

$$A = \begin{bmatrix} -2\alpha & \alpha & \alpha \\ \alpha & -2\alpha & \alpha \\ \alpha & \alpha & -2\alpha - \beta \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

and

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

with $\alpha = 1$ and $\beta = 0.5$.

There are nine possible configurations, but given the input and output matrices combinations are independent, it is sufficient to solve six LMIs (15.2) and (15.3) so as to obtain the following gains:

$$J_1 = \begin{bmatrix} 0.8018 & 1.0654 & -4.4610 \\ 0.8977 & -1.1775 & 1.4267 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad J_2 = \begin{bmatrix} 1.9252 & -5.4114 & 1.2044 \\ 0 & 0 & 0 \\ 1.1919 & 0.4867 & -1.3340 \\ 0 & 0 & 0 \end{bmatrix},$$

$$J_3 = \begin{bmatrix} 0.9773 & -4.0721 & 1.0690 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.7539 & 1.3342 & -1.4642 \end{bmatrix}, \quad L_1 = \begin{bmatrix} 0.8372 & 0 & 0 \\ -0.9527 & 0 & 0 \\ -0.8679 & 0 & 0 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 0 & -0.9527 & 0 \\ 0 & 0.8372 & 0 \\ 0 & -0.8679 & 0 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0 & 0 & -0.7796 \\ 0 & 0 & -0.7796 \\ 0 & 0 & 0.9233 \end{bmatrix}.$$

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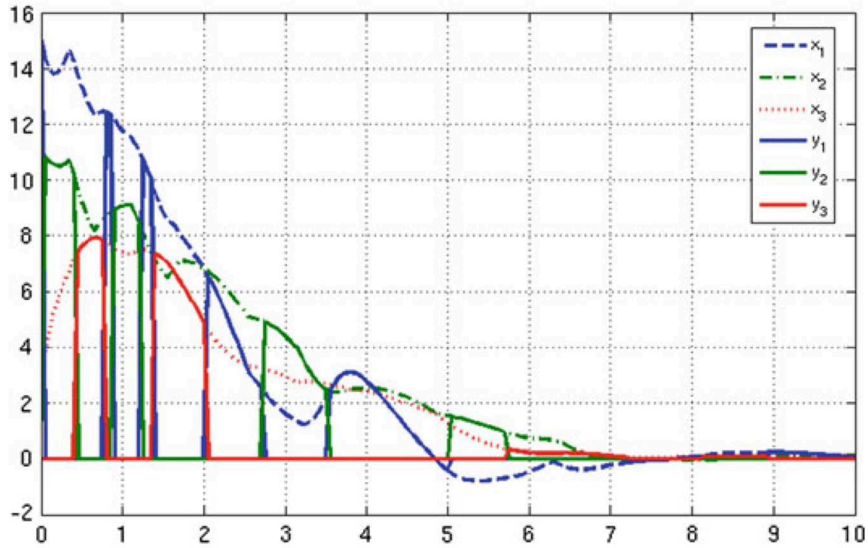

Fig. 15.3 Time histories of the tank level deviations from steady–state values

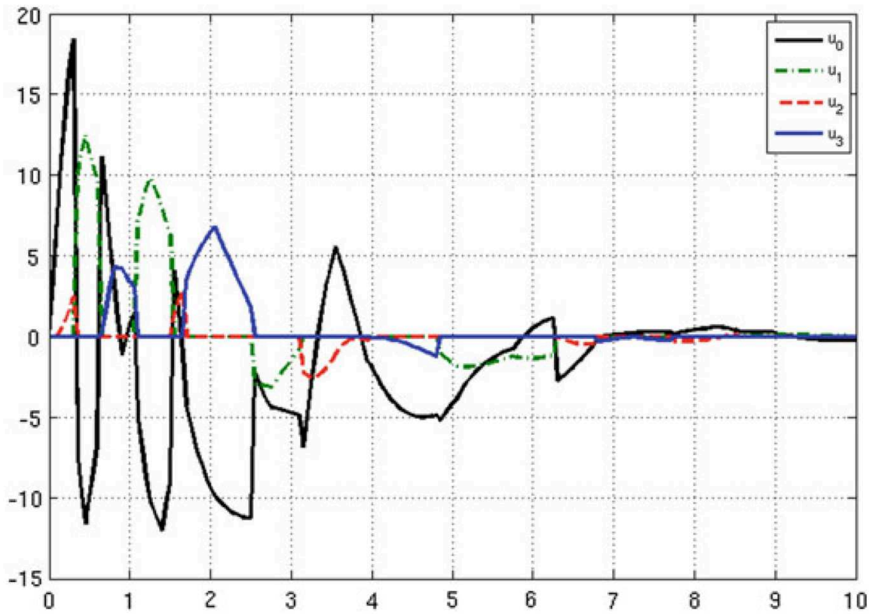

Fig. 15.4 Time histories of the flow deviations from steady–state values

During the system evolution, the ith sensor and the jth actuator faults are sensed and the corresponding input and output gains are used in the observer-based regulator (15.12).

Figures 15.3 and 15.4 depict the system signals evolution during the transient when the initial condition is $x(0) = [15\ 11\ 3]^\top$, an arbitrary sequence of faults occurs, and each of the Youla–Kucera parameters is set to zero (i.e., the standard Luenberger observer-based controllers are used).

To be more precise, Fig. 15.3 depicts the evolution of the state and actual output deviations from steady-state values. The representation in Fig. 15.3 has to be

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interpreted as follows: the solid line is the active output and the dotted lines are the state values. To clarify the representation, it can be noted that during the first instants, the second sensing channel is working, then the third, then the first just before $t = 1s$, then the second again, etc.

Figure 15.4 depicts the evolution of the system input deviation from the steady-state value, and the pictorial representation is as follows: the active output (corresponding to the working actuator) is working whenever it is different from zero. To make things clear, the second valve is active during the first instants, then the first, then the third, etc.

15.3.2 Discrete-Time Delays in Network Controlled Systems

Consider a network controlled n-dimensional discrete-time plant with m inputs and p outputs in the presence of unknown—but—bounded integer observation delay[3] $\tau(t) \in \{0, 1, \dots, \tau_{\max}\}$ (and without delays in the actuator channel). This system can be modeled by the equations

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t - \tau(t)), \end{aligned}$$

and can be alternatively represented by the following switching system, by adding delayed copies of the output to the state:

$$\begin{aligned} x_e(t+1) &= A^e x_e(t) + B^e u(t) \\ y(t) &= C^e_{\tau(t)} x_e(t) \end{aligned}$$

with $\tau(t) \in \{0, 1, \dots, \tau_{\max}\}$ and

$$A^e = \begin{bmatrix} A & 0^{n\times(\tau_{\max}-1)p} & 0^{n\times p} \\ C & 0^{p\times(\tau_{\max}-1)p} & 0^{p\times p} \\ 0^{(\tau_{\max}-1)p\times n} & I^{(\tau_{\max}-1)p} & 0^{(\tau_{\max}-1)\times p)} \end{bmatrix},$$

$$B^e = \begin{bmatrix} B \\ 0^{p\times m} \\ 0^{(\tau_{\max}-1)p\times m} \end{bmatrix},$$

$$C^e_0 = \begin{bmatrix} C \; 0^{p\times(\tau_{\max}-1)p} \; 0^{p\times p} \end{bmatrix},$$

$$C^e_i = \begin{bmatrix} 0^{p\times(n+(i-1)p)} \; I^p \; 0^{p\times(\tau_{\max}-i)p} \end{bmatrix},$$

[3]The delay is assumed to be a multiple of the sampling period if the discrete-time system is obtained as the discretization of a continuous-time system.

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for $i = 1, \ldots, \tau_{\max}$. The above system falls in the class of LPV systems described in the previous sections. Hence, assuming the satisfaction of the LMIs (15.4) and (15.5), it is possible to parametrize every stabilizing observer by means of the proposed Youla–Kucera structure. This in turn allows the designer to choose $\tau_{\max} + 1$ compensators, each stabilizing the system for a constant value of the delay τ, and realize such compensators by means of a proper Youla–Kucera parameter, thus guaranteeing stability of the system in the presence of arbitrary (bounded) sequences of delays. Note that the number of variables of the augmented system may be large if $\tau_{\max}$ is large, which may result in a heavy computational load. In addition, when the variation of the delay is very fast, the system may not be robust. We do not consider these questions here, since an analysis of the robustness of the method and of its efficiency when applied to a real problem is beyond the scopes of the chapter. For numerical examples of network controlled systems, we refer the reader to [18], or to [17].

15.3.3 Smith Predictor for LPV Stable Plants

The combination between time-delay and parameter-dependency may lead to several different linear parameter-varying time-delay systems (see, for instance, [10]). Here we focus on the problem of controlling an LPV stable continuous-time system in the presence of a known, time-varying but bounded, delay. We show that this problem can be solved with the technique described in Sect. 15.2. The considered dynamics are

$$\dot{x}(t) = A(w)x(t) + B(w)u(t),$$
$$y(t) = C(w)x(t - \tau(w)),$$

with $\tau(w) \in [0, \bar{\tau}]$, for some $\bar{\tau} > 0$, for all $w \in \mathcal{W}$. We denote by $\Pi(w) = \{A(w), B(w), C(w)\}$ the state-space representation of the delay—free part of the plant, by $W_P(s, w)$ its transfer function, by $W_C(s, w)$ the (parametric) transfer function of the controller and by $\Delta(s, w)$ the block corresponding to the delay, namely $\Delta(s, w) = e^{-\tau(w)s}$. These notations are used in the block diagram of Fig. 15.5 that represents the Smith predictor scheme.

Apart from the block associated with the delay, this scheme is analogous to the scheme depicted in Fig. 15.1, with the negative feedback loop inside the smaller dashed rectangle playing the role of the Youla–Kucera parameter; however, the block corresponding to $J(w)$ is absent, since the plant is assumed to be stable. The (parametric) transfer function of the Youla–Kucera parameter is

$$W_{YK}(s, w) = [I + W_C(s, w)W_P(w, s)]^{-1} W_C(s, w). \tag{15.16}$$

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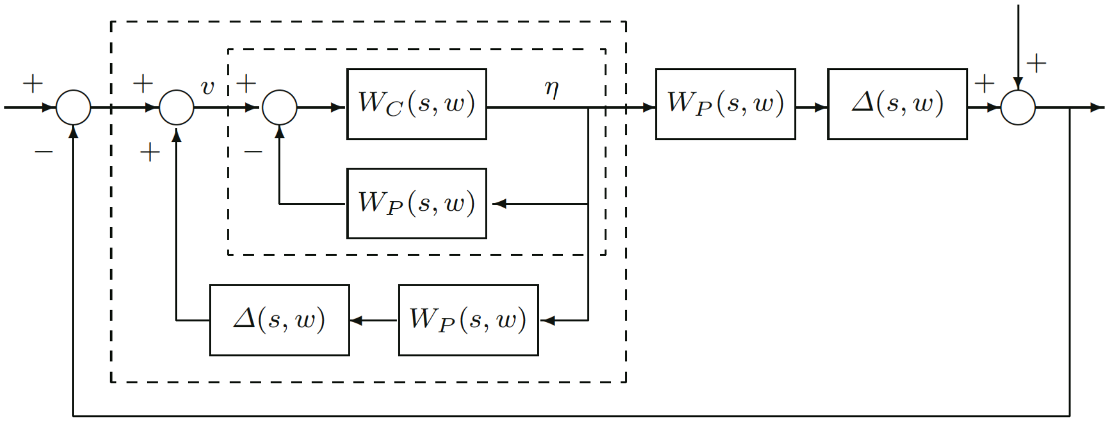

Fig. 15.5 Smith predictor control scheme

Consider, now, a state-space realization $\Theta(w) = \{F(w), G(w), H(w), K(w)\}$ of the transfer function (15.16) associated with the equations

$$\dot{z}(t) = F(w(t))z(t) + G(w(t))v(t),$$
$$\eta(t) = H(w(t))z(t) + K(w(t))v(t).$$

The overall system equations, in the absence of an external input, are

$$\dot{x}(t) = A(w(t))x(t) + B(w(t))u(t), \quad (15.17)$$
$$y_0(t) = C(w(t))x(t), \quad (15.18)$$
$$\dot{x}_c(t) = A(w(t))x_c(t) + B(w(t))u(t), \quad (15.19)$$
$$y_{c0}(t) = C(w(t))x_c(t), \quad (15.20)$$
$$\dot{z}(t) = F(w(t))z(t) + G(w(t))v(t), \quad (15.21)$$
$$\eta(t) = H(w(t))z(t) + K(w(t))v(t), \quad (15.22)$$
$$v(t) = y_{c0}(t-\tau) - y_0(t-\tau), \quad (15.23)$$

where x denotes the plant state, x_c denotes the state of the copy of the plant in the feedback loop and z denotes the state of the Youla–Kucera block Θ.

The main result concerning the stability of LPV control systems with delay is the following [6].

Theorem 15.3 *The control system (15.17)–(15.23), as in Fig. 15.5, is LPV stable if the state-space realization $\Theta(w)$ is LPV stable.*

In order to apply Theorem 15.3, one needs an LPV stable realization of the controller that can be obtained by steps 5 and 6 of the procedure reported in the Appendix.

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Remark 15.2 The scheme based on the Smith predictor can be fragile with respect to delay uncertainties (see [19]). However, we stress that in the present setting the time-varying parameters (one of which is the delay) are assumed to be exactly known at each time instant. A discussion on the robustness of the proposed method with respect to uncertainties in the plant only can be found in [6].

15.3.3.1 Example

As an example, we consider the dam–gallery system analyzed in [8] and we focus on the transfer function between the upstream flow Q_U and the downstream flow Q_D. By using simplified Saint-Venant's equations, and choosing the downstream flow as scheduling parameter, this transfer function turns out to be, [8],

$$W_P(w, s) = \frac{e^{-s\tau(w)}}{1 + k_1(w)s + k_2(w)s^2}, \tag{15.24}$$

where $w = Q_D$ and where $k_1(w)$, $k_2(w)$ and $\tau(w)$ are polynomials in w of degree three. A stability analysis, omitted for brevity, shows that, with the polynomial coefficients reported in [8], system (15.24) is LPV stable. Therefore, Theorem 15.3 can be applied. The LPV controller

$$R(s, w) = K\frac{1 + k_1(w)s + k_2(w)s^2}{s(1 + sT)},$$

for instance, which cancels the system dynamics and introduces the new poles 0 and $-1/T$, can be adopted. If $R(s, w)$ is realized according to steps 4, 5, and 6 of the procedure reported in the Appendix, then the closed-loop system is stable for any time-varying w.

15.3.4 Youla–Kucera Parameter as an Input Limiter for Constrained Systems

Another interesting problem is the control of LPV systems with input saturation (see, for instance, [11, 34]). Here we propose a solution based on the idea that, since the Youla–Kucera parameter can be any stable operator that maps the signal $o(t)$ to the signal $v(t)$, it can be exploited to achieve override control, [12]. When the absolute value of the control input is constrained to remain below a threshold $\bar{u}$, the principle of the override control is to consider the actual control input $u(t)$ as the sum of the ideal stabilizing control $u_{stab}(t)$ and of an additional signal $v(t)$ defined by

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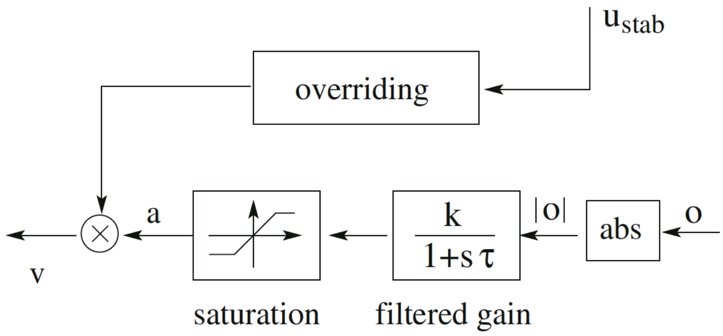

Fig. 15.6 The overriding Youla–Kucera block

$$v(t) = \begin{cases} \bar{u} - u_{stab}(t) & \text{if} \quad u_{stab}(t) > \bar{u} \\ 0 & \text{if} \quad |u_{stab}(t)| \leq \bar{u} \\ -\bar{u} - u_{stab}(t) & \text{if} \quad u_{stab}(t) < -\bar{u} \end{cases}$$

thus guaranteeing $|u(t)| \leq \bar{u}$. Since the actual control is different from the ideal one, the problem is how to ensure stability of this type of scheme. A possible solution, justified by the fact that the override control is useful at the beginning of the transient, is to activate it only when the absolute value of the observer error is above a given threshold $\bar{o}$. When $|o(t)| < \bar{o}$, the state is suitably reconstructed and the override signal is inactivated. A realization of this idea is depicted in Fig. 15.6. The absolute value of $o(t)$ is filtered by a filter with transfer function $\frac{k}{1+s\tau}$ and then saturated to 1 to obtain the activation signal $a(t)$. The aim of the filter is twofold. First, it avoids the action to be disabled if the signal $|o(t)|$ is less than the threshold for a too short interval (for instance when it "passes through zero"). Second, the constant gain k can be chosen so as to scale the estimation error to a magnitude suitable for the saturation function. For large values of $o(t)$, $a(t) = 1$ and the override control is active; after the transient, $o(t)$ goes to zero and so does $a(t)$.

Figs. 15.7 and 15.8 report the transient for the system with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \tag{15.25}$$

and $\bar{u} = 1$. The matrices provided by the LMIs (15.2) and (15.3) are

$$J = \begin{bmatrix} -1 & -2 \end{bmatrix}, \quad L = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

The parameters of the filter are $k = 10$ (corresponding to $\bar{o} = 0.1$) and $\tau = 3$. Figure 15.7 shows the transient from the initial condition $[2 \;\; 2]^\top$ without the overriding scheme: it can be seen that the control bound is deeply violated. Conversely, Fig. 15.8 shows the transient from the same initial condition $[2 \;\; 2]^\top$ with the overriding scheme: control bounds are not violated. The scheme can be applied in the same way also when observer and feedback gain are gain–scheduled.

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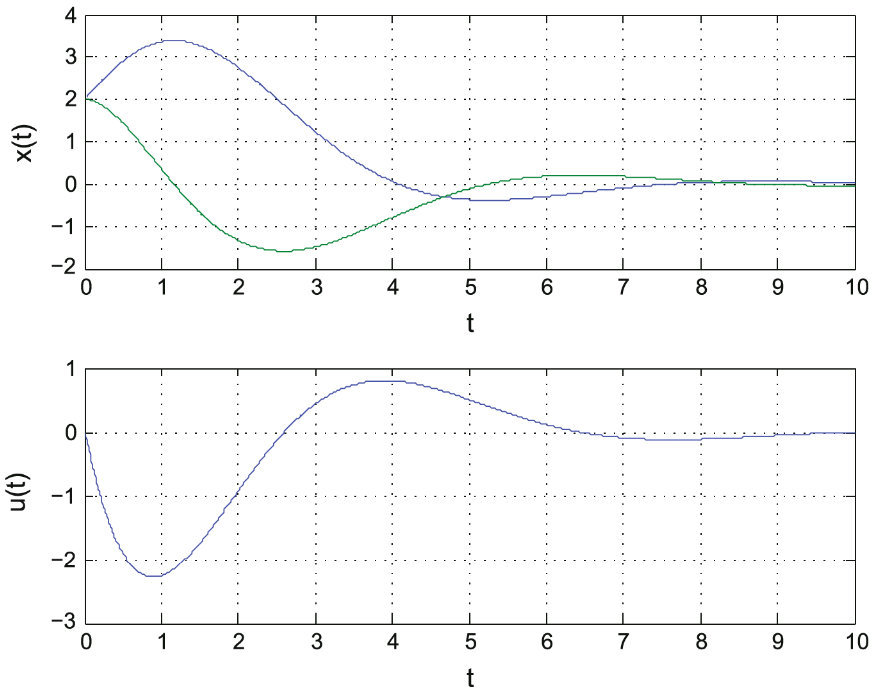

Fig. 15.7 The transient of the states (*top*) and of the control input (*bottom*) for the system (15.25) without the overriding scheme

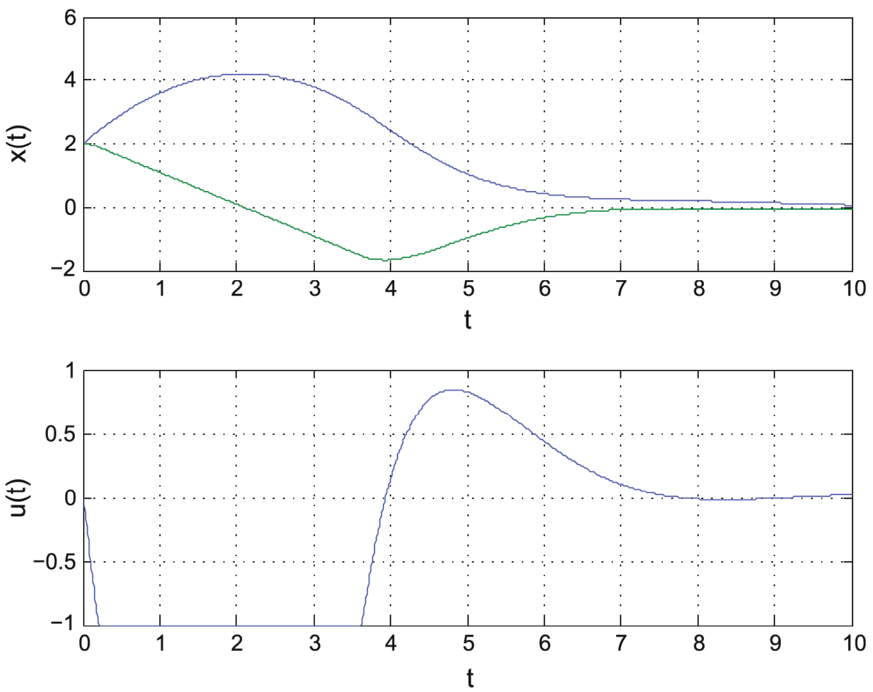

Fig. 15.8 The transient of the states (*top*) and of the control input (*bottom*) for the system (15.25) with the overriding scheme

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15.4 Conclusions

In the present work, the class of LPV systems and some recent fundamental results, concerning the possibility of resorting to the classical Youla–Kucera parametrization of stabilizing controllers, have been introduced. The presented results have been accompanied by several case studies, to illustrate the potential benefits of the proposed approach in different areas. Future research directions include a full characterization of override control schemes in an LPV framework, an investigation of the benefits of the LPV approach in fault-tolerant multisensor control schemes, [21], and the exploitation of recent set-theoretic results in the time-delay framework, [32], to provide a solution to the control of LPV continuous-time open-loop unstable plants in the presence of time-varying bounded delays.

Appendix

Proof of Theorem 15.2

Proof We sketch the proof of the first part, and of the second part for the case in which the operator is linear. The interested reader is referred to [4] for the general case.

Consider the variables $e(t) = \hat{x}(t) - x(t)$, $x(t)$ and $z(t)$, so that the resulting dynamic system is

$$\begin{aligned}\sigma(e) &= [A(w) + L(w)C(w)]e\\ \sigma(x) &= [A(w) + B(w)J(w)]x + B(w)J(w)e + B(w)v\\ \sigma(z) &= g(z, o, t)\\ o &= Ce\end{aligned}$$

In view of the quadratic stabilizability conditions in Theorem 15.1, both matrices $A(w) + L(w)C(w)$ and $A(w) + B(w)J(w)$ are asymptotically stable (since each of them admits a single quadratic Lyapunov function). Hence, the variable $e(t) \to 0$ as $t \to \infty$. This in turn implies that also $o(t) \to 0$ and, since the operator that maps the output error o into v is asymptotically stable, also $v \to 0$. Now, going back to the state evolution, this is governed by an asymptotically stable system fed by two signals that vanish as $t \to \infty$, which is enough to conclude that $x \to 0$.

As far as the converse part is concerned, assume there exists an LPV stabilizing regulator

$$\begin{aligned}\sigma(q(t)) &= F(w)q(t) + G(w)y(t)\\ u(t) &= H(w)q(t) + K(w)y(t)\end{aligned} \tag{15.26}$$

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so that the closed-loop system

$$\begin{bmatrix} \sigma(x) \\ \sigma(q) \end{bmatrix} = \begin{bmatrix} A(w) + B(w)K(w)C(w) & B(w)H(w) \\ G(w)C(w) & F(w) \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} = A_{cl}(w) \begin{bmatrix} x \\ q \end{bmatrix}$$

is LPV stable and admits a quadratic Lyapunov function. This means that there exists

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^\top & P_{22} \end{bmatrix} \succ 0$$

such that (in the continuous-time case), the following Lyapunov inequality is satisfied

$$A_{cl}(w)P + PA_{cl}^\top(w) \prec 0. \tag{15.27}$$

Denoting by $U(w) \doteq K(w)C(w)P_{11} + H(w)P_{12}^\top$, the upper–left block of (15.27) gives

$$\begin{aligned} &(A(w) + B(w)K(w)C(w))P_{11} + (B(w)H(w))P_{12}^\top \\ &\quad + P_{11}(A(w) + B(w)K(w)C(w))^\top + P_{12}(B(w)H(w))^\top \\ &\quad = A(w)P_{11} + P_{11}A(w)^\top + B(w)U(w) + U(w)^\top B(w) \prec 0, \end{aligned}$$

which corresponds to (15.2) with $P = P_{11}$. Similarly, if we pre- and post-multiply (15.27) by $P^{-1} = Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^\top & Q_{22} \end{bmatrix}$, we obtain $QA_{cl}(w) + A_{cl}^\top(w)Q \prec 0$ and, by considering the upper-left block of this expression, condition (15.3) with $Q = Q_{11}$ is obtained for $Y(w) \doteq Q_{11}B(w)K(w) + Q_{12}G(w)$. To conclude, it must be shown that the stabilizing LPV regulator can be realized as (15.6) and (15.13) for a proper stable operator (15.14). To this aim, set

$$\begin{aligned} J(w) &= U(w)P_{11}^{-1} \\ L(w) &= P_{22}^{-1}Y(w) \end{aligned}$$

and consider the observer-based stabilizing regulator (15.6). By resorting to the standard Youla–Kucera parameterization, since for every fixed value of w the resulting closed-loop system is stable, it is known that for each value of w the stabilizing regulator (15.26) can be realized as (15.6) and (15.13) where the stable operator (15.14) is linear, say

$$\begin{aligned} \sigma(z(t)) &= F_o(w)z + G_o(w)o(t) \\ v(t) &= H_o(w)z + K_o(w)o(t) \end{aligned} \tag{15.28}$$

Since the matrices $F_o(w)$ are Hurwitz stable (Schur stable, in the discrete-time case), each of them satisfies the Lyapunov equation

$$F_o(w)^\top P(w) + P(w)F_o(w) = -I \tag{15.29}$$

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for $P(w) \succ 0$. Now, let $\Omega(w)$ be the square root of $P(w) = \Omega^\top(w)\Omega(w)$ and apply, for each w, the following similarity transformation:

$$\begin{aligned} \tilde{F}_o(w) &= \Omega(w)F_o(w)\Omega(w)^{-1}, & \tilde{G}_o(w) &= \Omega(w)G_o(w), \\ \tilde{H}_o(w) &= H_o(w)\Omega^{-1}(w), & \tilde{K}_o(w) &= K_o(w) \end{aligned} \tag{15.30}$$

Notice that, in view of the applied transformation, all of the matrices $\tilde{F}_o(w)$ share the same quadratic Lyapunov function with $\tilde{P} = I$, since

$$\tilde{F}_o(w)^\top + \tilde{F}_o(w) = -\Omega^{-\top}(w)\Omega^{-1}(w) \prec 0$$

for every w. This amounts to saying that the Youla–Kucera parameter

$$\begin{aligned} \sigma(z(t)) &= \tilde{F}_o(w)z + \tilde{G}_o(w)o(t) \\ v(t) &= \tilde{H}_o(w)z + \tilde{K}_o(w)o(t) \end{aligned} \tag{15.31}$$

has the same input–output behavior as the "original" one and is LPV stable. □

The constructive proof described above for the determination of the Youla–Kucera parameter is summarized in the next procedure.

Given the LPV plant (15.1) and a family of LPV regulators of the form (15.6)–(15.14), each stabilizing the LPV plant for a constant value of w:

1. solve the LMIs (15.2) and (15.3) (or (15.4) and (15.5) in the discrete-time case);
2. compute the gains (15.7) and (15.8);
3. compute the Luenberger observer-based controller (15.6);
4. for every stabilizing regulator (15.6)–(15.14), compute the Youla–Kucera parameter (15.28);
5. solve the Lyapunov equation (15.29) (or the discrete-time Lyapunov equation, in the discrete-time case) and, for every w, determine the corresponding square root $\Omega(w)$ (such that $P(w) = \Omega^\top(w)\Omega(w)$);
6. apply the suggested transformation to derive the LPV stabilizing Youla–Kucera parameter (15.31).

Note that the described procedure has to be repeated for all $w \in \mathcal{W}$.

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